mean diameters of the inner recuperator tube; d_3, the inside diameter of the outer recuperator tube; L, the length of the heat-exchange section; F, the area of the heat-exchange surface; T, the temperature of the heat carrier; ΔT_{av}, the average temperature head; Q, the heat output; n, the number of heat-exchange sections; $Re = wd/\nu$, the Reynolds number; $Pr = \nu/a$, the Prandtl number; G, the heat-carrier flow rate; ΔP, the pressure drop. Subscripts: H, the magnetic field; 0, the absence of a magnetic field; 1, the first heat carrier; 2, the second heat carrier; a prime indicates the inlet to the recuperator; double prime indicates the outlet from the recuperator; ↑↑, direct motion of the heat carrier; ↑↓, counterflow of the coolant.

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CLASSIFICATION OF THERMAL MODELS OF FLOWTHROUGH SYSTEMS OF THERMOSTATICALLY CONTROLLED OBJECTS AT VARIOUS TEMPERATURE LEVELS

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We describe an approach to the selection of simplified methods of designing flowthrough systems of thermostatically controlled objects at various temperature levels; this approach is based on the classification of thermal models of the system, in terms of the nature of the thermal links.

Frequent use is made in devices and industrial installations containing thermostatically controlled units of flowthrough thermostatic-control systems (STC) in which the coolant for heat carrier flows through heat exchangers in thermostatically controlled objects (OTC). Where necessary to maintain various temperature levels in OTC in close proximity to each other it is advisable to use sequential separation of the conduit with the coolant to ensure minimum coolant consumption.

In determining the requirements to be imposed on the parameters of such STC, we must take into consideration the mutual thermal effect of the OTC, thus limiting the applicability of the calculation method developed for thermostats designed to stabilize the temperature of a single object [1, 2].

It is the purpose of this paper to undertake the classification of the thermal models of systems to reflect the structural features of the indicated STC, and this classification is based on determination of the criteria of maximum and minimum coolant flow rate, as well as of the strong and weak thermal links between the elements, thus allowing us to make recommendations with regard to the simplification of the calculation methods. The research was carried out for the case of thermostatic control of three sequentially cooled objects; however, it is not difficult to extend both the results and conclusions to STC with an arbitrary number of OTC.

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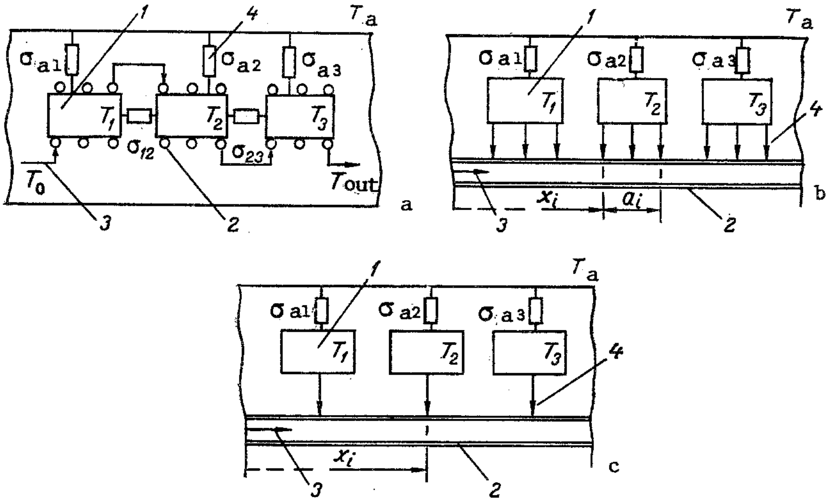

Fig. 1. Thermal STC models: 1) OTC; 2) conduit; 3) coolant flow; 4) thermal links.

The advantages of this approach can be realized most effectively in analyzing the steady-state thermal regime of isothermal OTC, whose unique feature is the analytical solution of the problem in a linear formulation that is primarily oriented to an STC type cryostatic control system (SCC), and it is also very effective in determining the general requirements imposed on the SCC parameters during the earlier design stage.

1. Thermal STC Models. In its most general form, the thermal model of an STC at three temperature levels can be illustrated by the diagram shown in Fig. 1a. The OTC is cooled by means of a coolant with an inlet temperature T_0, it is heated by heat flowing in from the ambient medium with a temperature T_a by means of conductivity σ_{ai}, and in the general case by means of internal sources of heat generation; the second temperature level, pertaining to the motion of the OTC coolant, is associated with the first and the third by the thermal conductivities σ_{12} and σ_{23}. All of the OTC can mutually affect each other through the mechanism of heat overflow through the conduit. There is no link between the first and the third OTC: $\sigma_{13} = 0$.

In the general formulation, the problem of the thermal analysis of the STC can be solved by numerical integration of a system of nonlinear differential equations which describe the temperature fields in the conduit, in the coolant, in the heat ridges, and in the heat OTC. This is an effective means suitable for forecasting the characteristics of completed designs.

For purposes of selecting a general plan and the STC parameters at an earlier stage in the design, the preferred approach is based on an analytical solution which requires the representation of the STC by means of a rather simple thermal model.

We will limit the range of STC to be considered to three particular circuits, which are nevertheless both typical and characteristic, and in each of these the thermal links between the elements are represented by concentrated parameters. The first circuit (model 1, Fig. 1a) corresponds to the case of an ideal thermal link between the OTC and the heat exchanger, i.e., $\sigma_i \to \infty$ (the temperature of the conduit at the segments in contact with the OTC coincides with the temperature of the OTC and is constant); the excess flow of heat through the conduit is negligibly small. Consideration is given to the radiative exchange of heat between the unloaded segment and the ambient medium. This circuit corresponds to a thermal model with concentrated parameters.

The second circuit (model 2, Fig. 1b) corresponds to the case of terminal thermal links σ_i and the absence of thermal links between the elements ($\sigma_{12} = \sigma_{23} = 0$). The thermal bridges from the OTC are in contact with the conduit over the entire side surface of the conduit at the end section a_i. The temperature field in the conduit is assumed to be uniform along its axis [3, 4], consideration being given to the possible radiative exchange of heat with the ambient medium. This circuit is described by a thermal model which contains distributed parameters.

The third circuit (model 3, Fig. 1c) is a special case of circuit 2 for the case in which $a_i \to 0$, which corresponds to some widely used means of heat removal (for example, thermal support-bearing interceptors [5, 6], cryostatic control of point links involving the use of bunched copper conductors, etc.).

2. The Model with Concentrated Parameters. The thermal OTC regime in model 1 is described by a system of three algebraic heat-balance equations:

$$\sigma_{a1}(T_a - T_1) + \sigma_{12}(T_2 - T_1) = Q_1 - P_{01},\quad \sigma_{a2}(T_a - T_2) - \sigma_{12}(T_2 - T_1) + \\ + \sigma_{23}(T_3 - T_2) = Q_2 - P_{02},\quad \sigma_{a3}(T_a - T_3) - \sigma_{23}(T_3 - T_2) = Q_3 - P_{03}. \tag{1}$$

The quantity of heat Q_i removed by the coolant is defined as the enthalpy difference:

$$Q_i = cM(T_{out\,i} - T_{0i}),\quad i = 1, 2, 3. \tag{2}$$

In the case of an isothermal heat exchanger, the coolant temperature $T_{out\,i}$ at the outlet is expressed in terms of T_{0i} by means of the following relationships [7]:

$$T_{out\,i} = T_{0i} + (T_i - T_{0i})E_i,\quad E_i = 1 - \exp(-\varphi_i),\quad \varphi_i = \frac{\alpha_i S_i}{cM},$$
$$T_{0i} = T_{out\,j} + \Delta T_{ji},\quad \Delta T_{ji} = \frac{P_{ji}}{cM},\quad T_{01} = T_0,\quad j = i - 1, \tag{3}$$

while the influx of heat P_{ji} to the nonworking segment of the conduit between the OTC is regarded either as given or it is assumed to be the flow of heat from the body to the tube.

When we take (2) and (3) into consideration, the terms in the right-hand side of system (1) have the form

$$Q_i - P_{0i} = cME_i(T_i - T_{0i}) - P_i,\quad T_{02} = T_0 + (T_1 - T_0)E_1,$$
$$T_{03} = T_0(1 - E_1)(1 - E_2) + T_1E_1(1 - E_2) + T_2E_2, \tag{4}$$

and it turns out that the temperature of coolant at the inlet to the i-th element may be regarded as equal to the temperatures at the outlet from the (1 − 1)-th element, i.e., $T_{01} = T_0$, $T_{02} = T_{out\,1}$, $T_{03} = T_{out\,2}$, if we combine the heat flowing into the nonworking segments of the conduits with the internal generation of heat in the OTC into total heat flows $P_i = P_{0i} + P_{ji}E_i$.

Having solved system (1), with consideration of (4), we find the expression for the temperature of the middle element:

$$T_2 = T_2(T_a, T_0) + T_2(P_i),\quad T_2 = \frac{T_a + DT_0}{1 + D},$$
$$D = \frac{cM[N_1 - \sigma_{a1}\sigma_{23}E_1(1 - E_2)E_3 - \sigma_{a1}A_3E_1E_2]}{N_2 + \sigma_{a1}A_3cME_1E_2 + \sigma_{a1}\sigma_{23}cME_1(1 - E_2)E_3},$$
$$T_2(P_i) = \frac{P_1[\sigma_{12}A_3 + cME_1E_2A_3 + \sigma_{23}cME_1E_3(1 - E_2)] + P_2A_1A_3 + P_3\sigma_{23}A_1}{N_2 + N_1cM}; \tag{5}$$
$$N_1 = A_1A_3E_1 + (\sigma_{12}A_3E_1 + \sigma_{23}A_1E_3 - \sigma_{12}\sigma_{23}E_1E_3)(1 - E_2),$$
$$N_2 = \sigma_{a1}\sigma_{12}A_3 + \sigma_{a2}A_1A_3 + \sigma_{a3}\sigma_{23}A_1,\quad A_i = \sigma_{ai} + \sigma_{i2} + cME_i,\quad i = 1, 3.$$

The quantities T_1 and T_3 are expressed in terms of T_0, T_{0i}, and T_2 according to the formula

$$T_i = \frac{1}{A_i}(\sigma_{ai}T_a + \sigma_{i2}T_2 + cME_iT_{0i} + P_i),\quad i = 1, 3. \tag{6}$$

Let us examine a typical and most interesting case, from the standpoint of actual practice, that is based on two limitations: 1. $E_i = 1$. In actual STC, when using effective heat exchangers, we find the realization of the conditions $\varphi_i > 2$-4, under which $E_i > 0.9$. Analysis of the solutions for (5) and (6) demonstrated that in the region $E_i = 0.8$-1 the

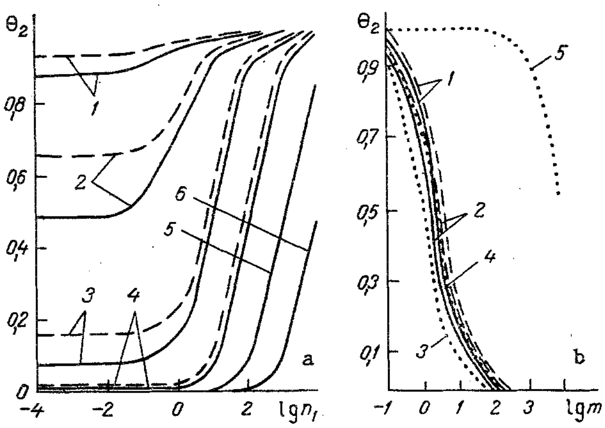

Fig. 2. Relative temperature of the second OTC as a function of n_1 and m: a) solid lines, for $n_3 = 10^{-4}$; dashed lines, for $n_3 = 10^4$; 1) m = 0.1; 2) 1; 3) 10; 4) 100; 5) 1000; 6) 10,000. b) Solid lines for $K_3 = 10^{-4}$, dashed lines for $K_3 = 10^4$; 1) $K_1 = 10^{-4}$; 2) $K_1 = 10^4$, the curves correspond to the case in which $n_1 = n_3 = 1$; the dashed curves represent $K_1 = K_3 = 1$: $n_1 = 10^{-4}$ for $n_3 = 10^{-4}$ (3) and $n_3 = 10^4$ (4); $n_1 = 10^4$ for $n_3 = 10^4$ (5).

values for the temperatures T_i change only slightly, which validates the introduction of $E_i = 1$. 2. $T_2(P_i) \ll T_2(T_a, T_0)$. This assumption is valid for systems of cryostatic control of passive objects to whose analysis this research is primarily directed. Examination of (5) demonstrates that for the case of uniform parameters $P_1 = P_2 = P_3 = P$, $\sigma_{a1} = \sigma_{a2} = \sigma_{a3} = \sigma_0$, $\sigma_{12} = \sigma_{23} = \sigma$, the most rigorous limitation on the quantity P, satisfying the criterion of smallness, is imposed for the case of a small relative flow rate ($cM \ll \sigma_0$, $cM \ll \sigma$) and is formulated in the form of the condition $\sigma_0 T_a \gg P$. In the case of strong thermal links between the elements ($\sigma \gg \sigma_0$, $\sigma \gg cM$) the criterion of smallness in P is formulated as $1/3cMT_0 + \sigma_0 T_a \gg P$, while with a large relative flow rate ($cM \gg \sigma_0$, $cM \gg \sigma$) it is formulated by the relationship $\sigma_0 T_a + \sigma T_0 \gg P$.

Introduction of these limitations enables us to eliminate the cumbersome aspects of the solutions for (5) and (6), and to write the expressions for the relative OTC temperatures in simple form in terms of the dimensionless parameters:

$$\Theta_2 = \frac{1 + \frac{n_3 K_3}{N_3} + \frac{K_1 + m}{N_1} n_1}{1 + \frac{n_3 K_3}{N_3} + \frac{K_1 + m}{N_1} (n_1 + m)}, \quad \Theta_j = \frac{T_j - T_0}{T_a - T_0}, \quad j = 1, 2, 3, \tag{7}$$

$$\Theta_1 = \frac{n_1}{N_1} + \frac{K_1}{N_1} \Theta_2; \quad \Theta_3 = \frac{n_3}{N_3} + \frac{K_3 + m}{N_3} \Theta_2, \quad N_i = n_i + K_i + m,$$

$$n_i = \sigma_{ai}/\sigma_{a2}, \quad K_i = \sigma_{i2}/\sigma_{a2}, \quad i = 1, 3, \quad m = cM/\sigma_{a2}.$$

Relationships (7) are extremely convenient for purposes of analyzing the influence exerted by the thermal links and by their uniformity on the thermal regime of the OTC. We are interested in examining initially the limit cases of strong and weak links between the OTC, i.e., the situation in which $K_i = 0$ of $K_i = \infty$; in this case, we obtain quite simple formulas for Θ_i and these are shown in Table 1. The case $K_1 = K_3 = \infty$ leads to a trivial model for the combining of three OTC into a single uniform element, and the only practical conclusion which follows from the analysis of these relationships boils down to the fact that effective cooling ($\Theta_1 = \Theta_2 = \Theta_3 \to 0$) is achieved when the conditions $cM \ll \sum_{i=1}^{3} \sigma_i$ are met. The cases $K_1 = 0$, $K_3 = \infty$ and $K_1 = \infty$, $K_3 = 0$ result in a slightly more complex model for two elements.

Of greatest interest is the limit case $K_1 = K_3 = 0$, in which we achieve the simplest regulation of the temperature levels for each of the three OTC as a consequence of using only three parameters: m, n_1, and n_3. In this case, in the case of effective cooling of

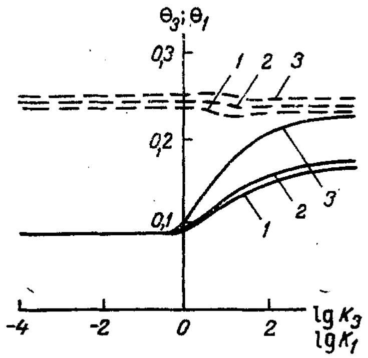

Fig. 3. The functions $\Theta_1(K_1)$, solid lines: 1) $K_3 = 0$; 2) 1; 3) ∞; $\Theta_3(K_3)$, dashed lines: 1) $K_1 = 0$; 2) 1; 3) ∞.

TABLE 1. Expressions for θ_i for Limit Values of K_i

K_1	θ_i	$K_3 = 0$	$K_3 = \infty$
0	θ_1	$\frac{n_1}{n_1+m}$	$\frac{n_1}{n_1+m}$
	θ_2	$\frac{1+\frac{n_1 m}{n_1+m}}{1+m}$	$\frac{1+n_3+\frac{n_1 m}{n_1+m}}{1+n_3+m}$
	θ_3	$\frac{n_3}{n_3+m}+\frac{m}{n_3+m}\theta_2(K_1, K_3=0)$	$\theta_2(K_1=0, K_3=\infty)$
∞	θ_1	$\frac{1+n_1}{1+n_1+m}$	$\frac{1+n_1+n_3}{1+n_1+n_3+m}$
	θ_2	$\theta_1(K_1=\infty, K_3=0)$	$\theta_1(K_1, K_3=\infty)$
	θ_3	$\frac{n_3}{n_3+m}+\frac{m}{n_3+m}\theta_2(K_1=\infty, K_3=0)$	$\theta_2(K_1, K_3=\infty)$

the first OTC ($\Theta_1 \ll 1$) when $m \gg n_1$ we find extensive possibilities of controlling the temperature level of the second element: from $\Theta_2 \simeq 1$ when $n_1 \ll m \ll 1$ to $\Theta_2 \simeq \Theta_1$ when $1 \ll n_1 \ll m$. Selection of the values of Θ_2 imposes a limitation on the range of variations in Θ_3 from $\Theta_3 \simeq 1$ for $n_3 \gg m$ to Θ_2 when $m \gg n_3$.

As regards the system of elements under consideration, we find interest in determining the criteria of the maximum and minimum values of the parameters, i.e., such of these quantities on the basis of which the limit transition may be regarded as attained and whose theoretical relationships can be simplified to the form shown in Table 1. Determination of these criteria essentially involves solution of the problem of classifying the OTC thermal models, which is the fundamental goal of this research. We will begin with an analysis of the influence exerted by the STC parameters on the thermal regime of the second OTC which, occupying an intermediate position, is found under more complex heat-exchange conditions.

Figure 2a shows the functions $\Theta_2(n_1)$ when $K_1 = K_3 = 1$ and the various values of m, with the quantities n_3 given for the extreme cases, virtually corresponding to 0 and ∞. We can see from the figure that Θ_2 is defined by the values of m and n_1, while the influence of n_3 may be regarded as secondary. The effect of the parameters K_1 and K_3 is insignificant, as we can see from Fig. 2b, which shows the curves of $\Theta_2(m)$ for the limit values of K_1 and K_3, virtually corresponding to 0 and ∞. For purposes of comparison, this same graph also shows the curves for the low and high values of n_1.

Analysis of relationships (7), partially illustrated by the curves of Fig. 2, shows that the quantities K_1, K_3, and n_3 may affect the temperature level of the second OTC under the condition $n_1 + m < 10$; in the opposite case, the thermal regime of the intermediate element is determined primarily by the relationships between the quantities m and n_1. Let us note that when $n_1 + m < 10$ some degree of effective cooling is possible under the conditions $n_3 K_3 < N_3$ and $m > n_1$.

The noted trends in the change in Θ_2 as a function of the parametric quantities are rather obvious: Θ_2 diminishes with increasing m and K_1 and increases as n_1, n_3, and K_3 increase.

We are particularly interested in investigating the influence exerted by the nature of the thermal link to the second OTC of the first and third objects on their temperature level. Figure 3 shows the theoretical relationships $\Theta_1(K_1)$ for three characteristic values of $K_3(0, 1, \infty)$ and $\Theta_3(K_3)$ when $K_1 = 0, 1, \infty$ for the case of uniform links between the OTC and the ambient medium ($n_1 = n_3 = 1$) and for the values of $m = 10$ at which the greatest possibilities are achieved for distributing the temperature levels of all three OTC. Based on the data of the figure, we can draw the conclusion that when K_1, $K_3 < 1$ a further reduction in the magnitude of K_i, all the way to zero, no longer exerts any effect on the temperature level of the first OTC, but it is only the quantity K_1 that exerts some influence on the third OTC and, on the whole, the thermostatically controlled elements may be regarded as insulated from one another. With K_1, $K_3 \to \infty$, and at magnitudes of $\sim 10^3$, the temperature levels of the first and third OTC asymptotically approach each other and all three OTC may be regarded as a single monolithic element. It should be noted that when K_1, $K_3 > 1$ the temperature of the OTC depends weakly on the links to the preceding object (for the third OTC), whereas the temperature of the first OTC depends markedly on the links to the subsequent OTC. However, with $K_3 < 1$ the temperature of the third OTC has some relationship to K_1, whereas Θ_1 in the case of $K_1 < 1$ is independent of K_3.

3. Models with Distributed Parameters. Having analyzed the circuits shown in Fig. 1b, c, we note that in both cases they are described by similar mathematical models and it is therefore useful to examine them in parallel. In this case, model 3 corresponds to the STC in which the excess of the thermally loaded sections of the heat exchanger are considerably smaller than the distances between them, and the change in conduit temperature in these sections is insignificant and need not be taken into consideration.

The mathematical model of the STC includes (Fig. 1b, c):

1. A differential equation for the conduit with boundary conditions that make provision for the fact that the temperature at the inlet to the conduit, as a rule, is known, or can be estimated, while the conduit itself is regarded as rather long:

$$\frac{d^2\Theta_w}{d\bar{x}^2} - b_{\text{in}}^2\Theta_w - b_{\text{ins}}^2(\Theta_w - \Theta_f) + Q(\bar{x}) = 0, \tag{8}$$

$$\Theta_w = T_w - T_a, \quad \Theta_f = T_f - T_a, \quad \Theta_w|_{\bar{x}=0} = \Theta_{w0}, \quad \left.\frac{d\Theta_w}{d\bar{x}}\right|_{\bar{x}=1} = 0,$$

$$b_{\text{in}}^2 = \sigma_{\text{rad}}/\sigma_{\text{t}}, \quad b_{\text{ins}}^2 = \sigma_{\text{con}}/\sigma_{\text{t}}, \quad \sigma_{\text{rad}} = \alpha_{\text{rad}} U_{\text{rad}} L, \quad \sigma_{\text{con}} = \alpha_{\text{con}} U_{\text{ins}} L, \quad \sigma_{\text{t}} = \lambda F/L,$$

$$\bar{x} = x/L.$$

The quantity $Q(\bar{x})$, with uniformly distributed inflows of heat to the heat exchanger from the OTC in the regions $[\bar{x}_i, \bar{x}_i + \bar{a}_i]$, is given in the form

$$Q(\bar{x}) = \sum_{i=1}^{N} \frac{P_i L^2}{\lambda F a_i} [U(\bar{x} - \bar{x}_i - \bar{a}_i) - U(\bar{x} - \bar{x}_i)],$$

$$\bar{x}_i = \frac{x_i}{L}, \quad \bar{a}_i = \frac{a_i}{L}, \quad U(\bar{x} - \bar{x}_i) = \begin{cases} 0, & \bar{x} < \bar{x}_i, \\ 1, & \bar{x} \geqslant \bar{x}_i, \end{cases}$$

while in the case of local heat inflows:

$$Q(\bar{x}) = \sum_{i=1}^{N} \frac{P_i L}{\lambda F} \delta(\bar{x} - \bar{x}_i),$$

where $\delta(\bar{x} - \bar{x}_i)$ is the Dirac function.

2. The differential equation which describes the distribution of temperature within the cryogenic flow, with boundary conditions of the first kind:

$$\frac{d\Theta_f}{d\bar{x}} - \varphi(\Theta_w - \Theta_f) = 0, \quad \varphi = \frac{\sigma_{\text{con}}}{cM}, \quad \Theta_f|_{\bar{x}=0} = \Theta_{f0}. \tag{9}$$

3. A system of two algebraic equations for cryostatically controlled elements, making it possible to associate the heat inflows P_i with the temperature distribution in the heat exchanger, which in the absence of intrinsic generation of heat in the OTC and a thermal link between the objects has the form

$$P_i = \sigma_i \left(\Theta_i - \Theta_w\Big|_{\substack{\bar{x}_i \leq \bar{x} \leq \bar{x}_i + \bar{a}_i \\ \bar{x} = \bar{x}_i}}\right), \quad P_i = -\sigma_{ai}\Theta_i, \quad \Theta_i = T_i - T_\rho,$$

whence

$$P_i = -\frac{\sigma_i \sigma_{ai}}{\sigma_i + \sigma_{ai}} \Theta_w\Big|_{\substack{\bar{x}_i \leq \bar{x} \leq \bar{x}_i + \bar{a}_i \\ \bar{x} = \bar{x}_i}}; \tag{10a}$$

$$\Theta_i = \frac{\sigma_i}{\sigma_i + \sigma_{ai}} \Theta_w\Big|_{\substack{\bar{x}_i \leq \bar{x} \leq \bar{x}_i + \bar{a}_i \\ \bar{x} = \bar{x}_i}}. \tag{10b}$$

Let us write the general solution of system (8), (9):

$$\Theta_w(\bar{x}) = \sum_{j=1}^{3} [A_j (B_j + D_j) + N_j \exp(\gamma_j \bar{x})], \tag{11}$$

where γ_1, γ_2, and γ_3 are the roots of a characteristic equation of the form

$$\gamma^3 + \varphi\gamma^2 - (b_{\text{ins}}^2 + b_{\text{in}}^2)\gamma - \varphi b_{\text{in}}^2 = 0,$$

$$A_1 = \frac{\gamma_2 - \gamma_3}{A}, \quad A_2 = \frac{\gamma_3 - \gamma_1}{A}, \quad A_3 = \frac{\gamma_1 - \gamma_2}{A},$$

$$A = \gamma_2\gamma_3(\gamma_3 - \gamma_2) - \gamma_1\gamma_3(\gamma_3 - \gamma_1) + \gamma_1\gamma_2(\gamma_2 - \gamma_1).$$

For the distributed inflows of heat

$$B_j = \sum_{i=1}^{N} \frac{P_i L^2}{\lambda F \bar{a}_i^2} \frac{\varphi(\Delta F - \Delta U)}{\gamma_i \bar{a}_i}, \quad D_j = \sum_{i=1}^{N} \frac{P_i L^2 \Delta F}{\lambda F \bar{a}_i},$$

$$\Delta U = U(\bar{x} - \bar{x}_i - \bar{a}_i) - U(\bar{x} - \bar{x}_i), \quad \Delta F = F_2 - F_1,$$

$$F_2 = \exp[\gamma_j(\bar{x} - \bar{x}_i - \bar{a}_i)]\, U(\bar{x} - \bar{x}_i - \bar{a}_i), \quad F_1 = F_2(\bar{a}_i = 0),$$

while for local heat inflows

$$B_j = -(\gamma_j + \varphi) \sum_{i=1}^{N} \frac{P_i L}{\lambda F} F_1, \quad D_j = 0,$$

N_j are constants determined from the boundary conditions and Eqs. (8) and (9):

$$N_3 = \frac{c_1\gamma_2 \exp(\gamma_2) - c_2\gamma_1 \exp(\gamma_1) - (\gamma_2^2 - \gamma_1^2) \sum_{j=1}^{3} A_j (B_j' + D_j')\gamma_j}{(\gamma_3^2 - \gamma_2^2)\gamma_1 \exp(\gamma_1) - (\gamma_1^2 - \gamma_3^2)\gamma_2 \exp(\gamma_2) + (\gamma_2^2 - \gamma_1^2)\gamma_3 \exp(\gamma_3)},$$

$$N_1 = \frac{c_2 + N_3(\gamma_3^2 - \gamma_2^2)}{\gamma_2^2 - \gamma_1^2}, \quad N_2 = -\frac{c_1 + N_3(\gamma_3^2 - \gamma_1^2)}{\gamma_2^2 - \gamma_1^2},$$

$$c_k = \Theta_{f0} b_{\text{ins}}^2 - \Theta_{w0}(b_{\text{ins}}^2 + b_{\text{in}}^2) + \Theta_{w0}\gamma_{\text{con}}^2, \quad k = 1, 2, \quad B_j' = B_j(\bar{x} = 1),$$

$$D_j' = D_j(\bar{x} = 1).$$

The comparatively cumbersome form of expression (11) with consideration of the structure of the parameters in this expression makes it necessary to undertake numerical calculations which involve iterations over formulas (11) and (10a) as well as requiring the subsequent calculations of Θ_i in accordance with formula (10b). These calculations are carried out on a computer with minimum expenditure of time and contain a minimum of initial information in compact form. The greatest advantages of this method become evident in the earliest stage of the design, in the selection of the SCC parameters through utilization of familiar optimization algorithms [8].

Significant simplification, making possible utilization of the derived theoretical relationships for analytical estimates, can be achieved if we assume the flow of heat from the ambient medium to the heat exchanger to be constant [9], i.e., $b_{in}^2\Theta_w$ = const, which is what actually occurs with a high degree of accuracy in the majority of cases. In this case, it turns out that $\gamma_3 = 0$ and

$$\gamma_{1,2} = -\frac{\varphi}{2} \pm \sqrt{\frac{\varphi^2}{4} + b^2_{\text{ins}}}, \quad \gamma_1 > 0, \quad \gamma_2 < 0. \tag{12}$$

For this case the derived solutions allow us to undertake simple analytical calculations of rather complex processes within the STC, and in particular we can determine the region of influence for the i-th source, i.e., the distance X from the point x_i of heat power input P_i, over which distance, with a given error δ_i, the entire flow of heat from the source is transmitted to the coolant:

$$\delta_i = \frac{P_t(X)}{P_t(\bar{x}_i)}, \quad P_t(X) = \lambda F \frac{d\Theta_w(X)}{dX}, \quad X = \bar{x} - \bar{x}_i, \quad P_t(\bar{x}_i) = P_i.$$

In view of the asymmetry of the processes, these have to be examined individually for the region $\bar{x} < \bar{x}_i$ and $\bar{x} > \bar{x}_i$; having carried out the appropriate operations, we can consequently obtain

$$X_i^- = \left| \frac{1}{\gamma_1} \ln \delta_i^- \right|, \quad X_i^+ = \frac{1}{\gamma_2} \ln \delta_i^+.$$

These formulas are convenient for purposes of estimating the distance X_i over which the sources virtually exert no influence on each other ($\delta_i \lesssim 0.1$) or, conversely, over which these sources can be combined ($\delta_i \gtrsim 0.9$). The ratio of the length of the zones of local heat-inflow influence in the direction of the coolant flow relative to the opposite direction (assuming $\delta_i^+ = \delta_i^-$) is equal to

$$\frac{X_i^+}{X_i^-} = \left| \frac{\gamma_1}{\gamma_2} \right| = \frac{\xi - 1}{\xi + 1}, \quad \xi = \sqrt{1 + \frac{4}{\varphi\psi}}, \quad \psi = \frac{\sigma_t}{cM},$$

where φ characterizes the relative quantity of heat conducted to the coolant flowing past; ψ denotes the relative conductivity of the conduit in the axial direction. When $\varphi\psi \gg 1$, i.e., where the intensity of the convective exchange of heat is extensive over a large heat-release area in the case of limited flow rates and in the presence of considerable thermal conductivity on the part of the conduit material, we find $X_i^+/X_i^- \approx 1/\varphi\psi \to 0$, while in the case of $\varphi\psi \ll 1$ the length of these zones are equal. It is the latter case that is most frequently encountered in actual practice, and it corresponds to the condition $b^2 \gg \varphi^2/4$ (when gaseous helium flows along a thin-walled copper conduit that is 5 mm in diameter and 1 m in length, this condition is normally present with a reserve of two orders of magnitude).

We used the proposed method as a basis for the calculation of a steady-state STC thermal regime in which three temperature levels are attained as a consequence of the local OTC thermal links to the heat exchanger; the difference in the results from a comparison of the OTC temperatures and the experimental data did not exceed 4%.

This investigations demonstrated the convenience of analytical calculations in solving the problem of classifying STC on the basis of large and small thermal-link criteria, as well as in determining the parameters which exert decisive and secondary influence. In many practical cases, the elements of the calculation methods based on models with concentrated and distributed parameters serve to complement each other at various stages of the analysis, making it possible to refine the initially adopted thermal model, thus facilitating the investigation into relatively complex STC circuits.

NOTATION

T_i, temperature of the i-th OTC, K; P_{0i}, power of the internal heat generation in the OTC, W; α_i, convective heat-transfer coefficient at the section in contact with the i-th OTC, $W/(m^2 \cdot K)$; S_i, surface area of convective heat exchange at the segment in contact with the i-th OTC, m^2; c, specific heat capacity of the coolant, $J/(kg \cdot K)$; M, mass flow rate of the coolant, kg/sec; T_w, temperature of the conduit wall, K; T_f, temperature of the coolant flow, K; α_{rad}, coefficient of radiative heat exchange between the conduit and the ambient medium, $W/(m^2 \cdot K)$; U_{rad}, outside perimeter of the conduit, m; L, length of the conduit, m; α_{con}, coefficient of convective heat exchange in the heat exchanger, $W/(m^2 \cdot k)$; U_{ins}, inside perimeter of the conduit, m; λ, coefficient of thermal conductivity for the material of the conduit, $W/(m \cdot K)$; F, cross-sectional area of the conduit, m^2.

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THE CONTROL OF QUASIUNIFORM HEATING OF A CYLINDRICAL SPECIMEN IN AN INDUCTOR

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We have used a computer mathematically to experiment with and to formulate solutions for the problem of optimum control by means of quasiuniform heating of Foucault currents in cylindrical steel specimens.

The technology used in the heat treatment of metal specimens requires that they be heated uniformly, within the limits of some tolerance δ, to a specified temperature $\hat{u}$. Such quasiuniform heating, regardless of its source, can either be achieved within some given interval of time, or within a minimum period of time whose estimation is of interest from the standpoint of economic control of the technological processes.

The problems of using quasiuniform heating to achieve control have been examined, in particular, in [1, 2], where the control functions where the temperature of the outside medium for the flow of heat coming from the outside was taken as a function of time.

In this paper, the heating is achieved by means of Foucault currents that are generated within the specimen by means of a high-frequency field from a solenoid inductor into which the specimen has been placed.

Within the framework of the axial-symmetric three-dimensional-uniform model in [3, 4] a method is applied to the problems of annealing steel specimens for purposes of calculating the temperature field generated by such a source.

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